

Assignment -V

MODULE-5

(IMPROPER INTEGRAL AND LAPLACE'S TRANSFORMATION)

1. Show that $\int_0^{\infty} e^{-4x} x^{3/2} dx = \frac{3}{128} \sqrt{\pi}$.
2. Show that $\int_0^{\pi/2} \sqrt{\tan x} dx = \pi/\sqrt{2}$.
3. Examine the convergence of the improper integral
 - a) $\int_0^2 \frac{dx}{x(2-x)}$
 - b) $\int_1^{\infty} \frac{dx}{x^2(x+1)}$
4. Find the Laplace transform of (a) $t e^{-2t} \sin 3t$ (b) $4t^2 - 3 \cos 2t + 5e^{-t}$.
(c) $t^3 e^{-3t}$ (d) $e^t \cos t \sin t$
5. Find the Laplace transform of (a) $\frac{e^{-3t}}{t} \sin 2t$ (b) $\frac{\cos 2t - \cos 3t}{t}$
(c) $\int_0^{\infty} t e^{-3t} \sin 2t dt$
6. Find $L\{G(t)\}$ where $G(t) = \begin{cases} (t-2)^3, & t > 2 \\ 0, & t < 2. \end{cases}$
7. If $L\{F(t)\} = \frac{s^2 - s + 1}{(s-1)(2s+1)^2}$, then prove that $L\{F(2t)\} = \frac{s^2 - 2s + 4}{4(s-2)(s+1)^2}$ by applying change of scale property.
8. Find the Laplace transform of (a) $\frac{e^{-3t}}{t} \sin 2t$ (b) $\frac{\cos 2t - \cos 3t}{t}$
9. Find the Laplace transform of $\int_0^{\infty} t e^{-3t} \sin 2t dt$
10. Find the value of the integral using Laplace Transform (a) $\int_0^{\infty} e^{-t} \sin 2t dt$ (b) $\int_0^{\infty} e^t \cos t dt$
11. Find $L(\frac{\sin at}{t})$. Hence show that $\int_0^{\infty} \frac{\sin t}{t} dt = \frac{\pi}{2}$.
12. Find $L\{\int_0^t e^t \cos 2t dt\}$.

13. Find $L\{F(t)\}$ where $F(t) = \begin{cases} \sin t, & 0 < t < \pi \\ \sin 2t, & \pi < t < 2\pi \end{cases}$

$$= \sin 3t, \quad t > 2\pi.$$

14. Find $\int_0^\infty te^t \cos t dt$

15. Evaluate $L\left\{\int_0^t \frac{e^{2t}}{t} \sin t dt\right\}$.

16. Evaluate $L\left\{\int_0^t te^{-3t} \cos 4t dt\right\}$.

17. Using Convolution find $L^{-1}\left\{ \frac{s^2}{(s^2+a^2)(s^2+b^2)} \right\}$

16. Solve the following differential equations

12. $y'' + 4y = \sin 2t$, given $y(0) = y'(0) = 0$.

13. $y'' - 2y' + 2y = 0$ $y = y' = 1$ at $x = 0$

14. $y'' - 2y' + x = e^{-t}$ $x(0) = 2$ $x'(0) = 1$

15. $y'' - y' - 2y = 20 \sin 2t$ given $y(0) = 0$ $y'(0) = 2$.

16. Show that $\int_0^\infty e^{-4x} x^{3/2} dx = \frac{3}{128} \sqrt{\pi}$.

17. Show that $\int_0^{\pi/2} \sqrt{\tan x} dx = \pi/\sqrt{2}$.

18. Examine the convergence of the improper integrals

$$\int_0^2 \frac{dx}{x(2-x)}$$

$$\int_1^\infty \frac{dx}{x^2(x+1)}$$