

Chapter 5

Surface tension

5.1 Definition

If a line element be imagined on the surface of a liquid in any direction, the surface tension of the liquid, at a given temperature, is defined as the force per unit length of the line element, acting normally to it and in the plane of the surface.

In the *C.G.S.* system the unit of surface tension is *dyne/cm* and in the *S.I.* system it is *newton/metre*.

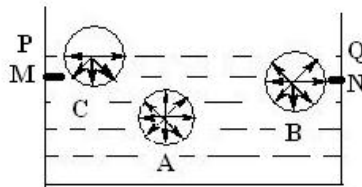


Figure 5.1: molecular interaction

Now consider three molecules A, B, C of a liquid. The molecule A is well inside the liquid, the molecule B is little below the free surface and the molecule C is just on the free surface. The sphere of influence of each molecule has been shown by a circle around it 5.1.

- Since the sphere of influence of the molecule A lies wholly inside the liquid, the molecule is attracted equally on all sides by the surrounding molecules lying within the sphere. Hence *no resultant cohesive force acts on the molecule A .*
- The sphere of influence of the molecule B , on the other hand lies partly outside and partly inside the liquid. As a result, the number of liquid molecules on the upper half of the sphere is less than that on the lower half and molecule B will experience a greater attractive force in the downward direction than in the upward. There will be a *resultant downward force on the the molecule B .*
- Since the molecule C is just on the free surface of the liquid, half of its sphere of influence lies outside the liquid. There is no molecule to exert an upward attractive force on the molecule C which will feel a resultant downward force perpendicular to the free surface of the liquid and in this case, the *force is maximum.*

If, now a plane MN be drawn parallel to the free surface PQ and at a distance equal to the molecular range from it, the layer of liquid lying in between the free surface PQ and the plane MN is called the surface film. All the molecules lying on the surface film experience a downward attractive force due to cohesion, the magnitude of which increases as the free surface PQ is approached. For this cohesive force, the molecules can not escape from the free surface; on the other hand, they have a tendency to enter into the liquid and thereby try to contract the free surface to the smallest possible area.

5.1.1 Surface energy

A thin wire is bent into right angles twice to give the framework $GBCH$. Another wire AD can slide without friction along the horizontal arms BG and CH (5.2). If the framework is dipped in soap solution and taken out, a thin film $ABCD$ will be formed in the space. As a result, surface tensional forces will act at every point on the wire AD , perpendicular to it and tangential to the surface of the film and the wire AD will slowly move towards the arm BC of the framework. To keep the wire AD fixed to its position, an equal force needs be applied on it in the opposite direction. If the surface tension of the liquid film be γ then, force per unit length of the wire AD in the direction as shown in the figure 5.2 is γ . If the length of the wire AD is ' l ', then the total force acting on the movable wire in the direction of the arm $BC = 2l\gamma$ (since the film has two surfaces- upper and lower- each producing a force $= l\gamma$)

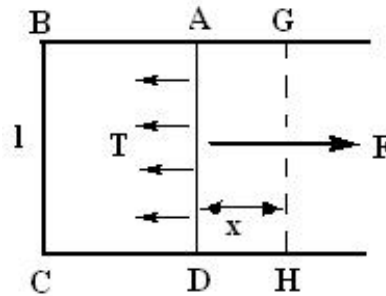


Figure 5.2: surface energy calculation

So, to keep the wire steady, the opposite force (F) that needs be applied on the movable wire AD is given by $F = 2l\gamma$. If the movable wire is moved a distance ' x ' perpendicular to its length to the position GH , the work done against the surface tension $= F \cdot x = 2l\gamma \cdot x$

The surface area of the film both upper and lower will, thereby, have an increment lx and the total increase in the surface area $= 2lx$.

Thus the work required to increase the surface area of the film by unity $= \frac{2l\gamma \cdot x}{2lx} = \gamma$

Similarly, if the surface area of a liquid is allowed to shrink by unity, then the work available is also γ .

This work remains stored in the film as its potential energy and is called the surface energy of the liquid film. So, surface tension of a liquid is numerically equal to the surface energy per unit area.

5.1.2 Excess pressure inside a spherical soap bubble

The bubble is nothing but some air enclosed by a spherical film of soap solution. Clearly the bubble has two surface - one inside and one outside. The molecules on both the surfaces will feel an inward force due to surface tension. The pressure inside the bubble is evidently greater than the pressure outside, otherwise the bubble will not be able to maintain its spherical shape. The

excess internal pressure tries to bulge out the bubble while the surface tensional forces try to minimize the surface area. When these two opposing forces balance, the bubble becomes stable. Let r = radius of a spherical bubble, P = excess internal pressure and γ = surface tension of soap solution.

Imagine the spherical bubble to be divided into two hemispheres by a plane (horizontal) ABC through its center. The following two forces act on the upper hemisphere.

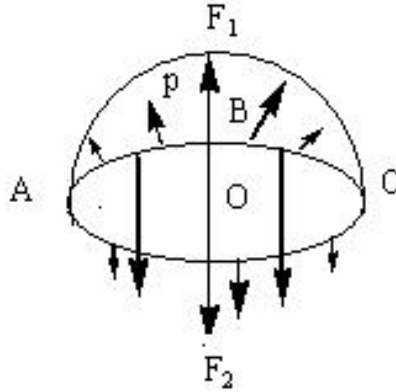


Figure 5.3: soapbubble

- An upward force on the plane ABC due to excess pressure P inside. Area of the surface $ABC = \pi r^2$. So, the upward force = $\text{pressure} \times \text{area} = \pi r^2 P$. Due to symmetry, this upward force acts through the center O of the bubble. In figure 5.3, F_1 represents this upward force.
- Downward forces along the circumference ABC due to surface tension. Now, the length of the circumference = $2\pi r$ and since the bubble has two surfaces, the total downward force F_2 , due to surface tension = $2 \times 2\pi r \gamma$. This force evidently acts through the centre O of the bubble in the downward direction. For the equilibrium of the bubble, we have

$$\begin{aligned} F_1 &= F_2 \\ \text{or, } 4\pi r \gamma &= \pi r^2 P \\ \text{or, } P &= \frac{4\gamma}{r} \end{aligned} \quad (5.1)$$

If the atmospheric pressure outside the bubble be P_{atm} , the total pressure inside the bubble,

$$P_{total} = P_{atm} + \frac{4\gamma}{r} \quad (5.2)$$

5.1.3 Excess pressure inside a liquid drop or an air bubble in water

When air bubbles are formed inside water or if we take small liquid drops, we are to consider only *one surface*. Unlike soap bubbles, air bubbles or liquid drops have only one surface. If r be the radius of such bubbles or drop, then, according to figure 5.3, the upward force due to excess pressure P is given by $F_1 = \pi r^2 P$ and the downward force due to surface tension $F_2 = 2\pi r \gamma$ (for one surface).

$$\begin{aligned} \text{Hence, for equilibrium, } \pi r^2 P &= 2\pi r \gamma \\ \text{So, } P &= \frac{2\gamma}{r} \end{aligned} \quad (5.3)$$

If the air bubble is formed at a depth h from the free surface of water and the atmospheric pressure be P_{atm} , then total inside pressure P is given by,

$$P = P_{atm} + h\rho g + \frac{2\gamma}{r} \quad (5.4)$$

where ρ is the density of water.

5.1.4 Contact angle

The angle between the tangent to the curved liquid surface at the point of contact and the solid surface, inside the liquid, is called the angle of contact for pair of solid and liquid.

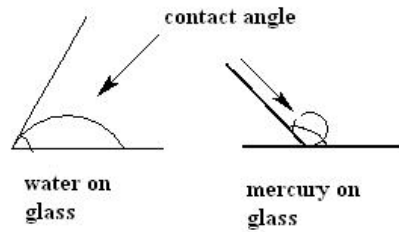


Figure 5.4: contact angle

We define contact angle as Θ . Θ is $< 90^\circ$ (acute angle) for wetting. The most ideal case of wetting is $\Theta = 0$. Wetting decreases as Θ increases. For a case where $\Theta > 90^\circ$, we say there is no wetting. On glass, contact angle = 140° for mercury and it is 8° for ordinary water.

5.1.5 Capillary action

If one end of a capillary tube be dipped vertically in water, the liquid will rise along the tube until it stands at some height above the level in the vessel. But if the same capillary tube is dipped in mercury (or any other liquid which does not wet glass), the mercury level in the tube goes below the level in the vessel, i.e. there is depression of mercury column in the tube. The ascent or descent of liquid in a capillary tube is known as capillarity or capillary action. It causes oil to rise along the wick of a lamp or subsurface water in the soil to the top layer and to some extent, in the roots and stems of plants.

Rise of liquid in a capillary tube: A capillary tube of circular bore is dipped vertically in a liquid of surface tension γ . The liquid rises in it and its meniscus is concave upward. If Θ be the angle of contact between the solid and the liquid, the surface tensional force will act tangentially inward per unit length of the line of contact between the liquid and the solid (5.5), and the direction of the force will make an angle Θ with the wall of the tube. The glass wall, along the line of contact with the liquid will feel a force in the direction of the tangent. According to Newton's third law, every action has an equal and opposite reaction. The glass wall, therefore exerts an equal and opposite force γ on the liquid meniscus. The reaction may be resolved into two components (i) $\gamma \cos \Theta$ vertically up and (ii) $\gamma \sin \Theta$ outward horizontally. Considering the whole meniscus, the horizontal components cancel out, while the vertical components are added up. The total vertical force on the liquid meniscus supports the column of liquid standing in the tube.

If r be the radius of the tube at the point of contact with the liquid meniscus, then the total vertical force = length of the meniscus \times force per unit length = $2\pi r \times \gamma \cos \Theta$

If h be the height of the liquid column up to the bottom of the meniscus, the volume of the cylindrical liquid column of height ' h ', = $\pi r^2 h$

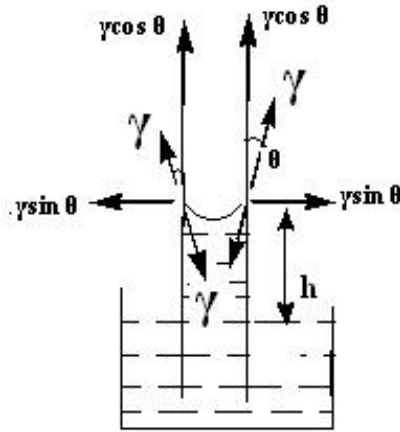


Figure 5.5: capillary action

and if v be the volume of the liquid in the curved portion, then, the total volume of the liquid raised = $\pi r^2 h + v$

The weight of this volume of liquid = $(\pi r^2 h + v)\rho \cdot g$

where ρ is the density of the liquid. Therefore,

$$\begin{aligned} 2\pi r \cdot \gamma \cos\Theta &= (\pi r^2 h + v)\rho \cdot g \\ \text{or, } \gamma &= \frac{(\pi r^2 h + v)\rho \cdot g}{2\pi r \cos\Theta} \end{aligned} \quad (5.5)$$

If v be negligibly small compared to the volume $\pi r^2 h$, then

$$\begin{aligned} \gamma &= \frac{(\pi r^2 h)\rho \cdot g}{2\pi r \cos\Theta} \\ &= \frac{r h \rho g}{2 \cos\Theta} \end{aligned} \quad (5.6)$$

5.2 Few things you need to know

- **Cohesive force:** Force of attraction/ repulsion between the molecules of the same substance is called as cohesive force. The greatest distance at which molecules can attract each other is called *molecular range*. Taking the position of a molecule as centre, if a sphere be imagined to be drawn with molecular range as radius, the sphere is called the *sphere of influence*.
- **Adhesive force:** Force of attraction between the molecules of different substances is called as adhesive force.
- In general the intermolecular force is of the order of 10^{-11} N where as gravitational force is 10^{-50} N. Intermolecular distances are of the order of 10^{-10} m

5.3 Variation of surface tension with temperature: Eotvos equation

The surface tensions of liquids generally decrease with increase of temperature. An empirical relation between surface tension and temperature was developed by Eotvos. According to this relation, if V be the molar volume of a liquid, and T_c the critical temperature of a liquid the surface tension γ is given by

$$\gamma V^{\frac{2}{3}} = k(T_c - T) \quad (5.7)$$

Where k is constant for all liquids.